Big Numbers

An article about card shuffling by the aptly named Card Colm has been making the rounds of the internet lately. In it, he vividly compares the number of unique ways a deck of 52 cards can be ordered with numbers that we can grasp.

At the end he says "You too can be unique. Just keep shuffling. You'll get there."

So I was thinking – If I shuffle a deck, what exactly are the chances that there has never been another shuffled deck of cards like that in the history of mankind? Well, we need two pieces of information to figure this out. The first is – how many ways are there to arrange a deck of 52 cards? This one is easy to figure out, even if it is a big number. It's 52x51x50x49, etc, all the way down to 1. This has a short name called 52-factorial, and it's written as a 52 with an exclamation point after it. 4!, for example is 4x3x2x1, or 24.

Factorials get really big, really fast. So big that 52 factorial is about $8x10^{67}$ – or 8 with 67 zeroes after it. That's a staggeringly large number. The number of atoms in the observable universe is estimated to be about 10^{80}, which is not that much bigger in the grand scheme of things – only about a trillion times bigger.

The other piece of information we need is – How many shuffles have occurred in the history of man? Now obviously we can't exactly determine this. So we need to fall back on our friend the estimate. Some people never see a deck of cards in their lives. Some people sit at a blackjack table and see lots of shuffled decks during a day. I'd like to err on what I think is the high side, so let's say that on average, one deck of cards is shuffled per person on earth per day. So if there are 7 billion people on earth, that's 7 billion shuffles per day, or 2.5 trillion shuffles per year.

But that's just now, when there are 7 billion people. The modern deck of cards goes back to about 1500, when there were less than a billion people. So we need to figure out how many total person-years there

have been, and then convert that to days. Because I am an insane crazy person I got a variety of statistics about the estimated world population from year to year. So adding everything together, there have been about 850 billion person-years from 1500 until 2012. Multiply by 365 and you get a tad over 300 trillion person days. So that's 300 trillion shuffles since the invention of the card deck. I really think this is too high, considering it includes every single man, woman, and child on all continents, since the year 1500, but let's go with 300 trillion for now.

So 300 trillion is expressed in scientific notation as 3×10^{14}. That's 3 with 14 zeroes after it. Remember that our deck of cards can be arranged in 8×10^{67}. So the total number of shuffles in history is a really, really small fraction of the total number of shuffles. If you shuffle a deck right now, the chances it is the same as one of those 300 trillion shuffles is 1 in 10^{53} – That's 0.0000000000 – 53 zeroes – and then 1. Very, very small.

So Mr. Colm's assertion that 'you can be unique – just keep shuffling and you'll get there' is not quite right. A single shuffle should be more than enough.

The fact that combining small numbers of things can give you a crazy number of combinations is really important in science. Combining just 52 things gave us almost as many combinations as the number of atoms in the observable universe.

For example, one of the theories of the origin of life is based around this concept. Here's the idea – Let's say you have a chemical reaction that turns A into B. That proceeds at a certain rate. But there are certain molecules that help turn A into B much faster. These are called catalysts. Our bodies are filled with catalysts, mostly proteins, that help turn things into other things faster than they would do spontaneously.

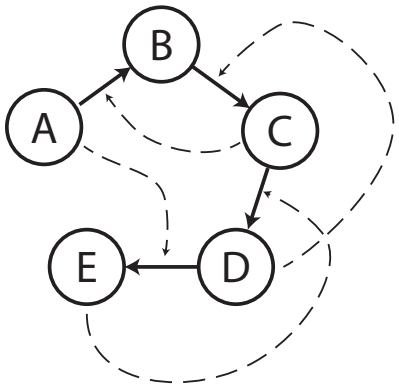
Now if I have a reaction that turns A into B, and just throw a random molecule C in there, it is very unlikely that C will act as a catalyst. Really, really unlikely. But if I have a whole bunch of different molecules forced into the same area, say, an undersea volcanic vent, the number of different combinations of molecules will grow very quickly. And the number will get much bigger than the chance that there is a molecule that will be a catalyst for two others.

And ultimately you may get a combination that loops back onto itself – So C helps turn A into B, and B helps turn D into E, and D helps turn C into A – it's hard to explain without a picture *(which I can now include, below)*, but basically a chain of reactions that loops back onto itself, so that the molecules help make more and more of each other.

This is what's called an *autocatalytic set*, and the theory says that once you get an autocatalytic set, you have ignition on the engine of life. And while the chances of a specific set of molecules being autocatalytic is really, really small, the number of such sets grows incredibly quickly with each additional type of molecule you have, the same way that adding more and more cards to a deck makes the number of possible arrangements incredibly large. Eventually the number of combinations of molecules gets so big that it becomes impossible that an autocatalytic loop won't develop.

If you're more interested in these ideas, admittedly still very hypothetical, check out the works of Stuart Kaufman, especially At Home In The Universe.

Who knew that the key to life's origins might lie in a deck of cards?



An unrealistically simple autocatalytic set. The solid lines show transitions between molecules. The dashed lines show which molecule helps that transition. So C helps A turn into B, and A helps D turn into E.